



## Cambridge International AS & A Level

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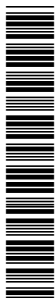
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**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1

**May/June 2023**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

- 1 The equation of a curve is such that  $\frac{dy}{dx} = \frac{4}{(x-3)^3}$  for  $x > 3$ . The curve passes through the point (4, 5).

Find the equation of the curve. [3]

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- 2 The coefficient of  $x^4$  in the expansion of  $(x + a)^6$  is  $p$  and the coefficient of  $x^2$  in the expansion of  $(ax + 3)^4$  is  $q$ . It is given that  $p + q = 276$ .

Find the possible values of the constant  $a$ .

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- 3 (a) Express  $4x^2 - 24x + p$  in the form  $a(x + b)^2 + c$ , where  $a$  and  $b$  are integers and  $c$  is to be given in terms of the constant  $p$ . [2]

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- (b) Hence or otherwise find the set of values of  $p$  for which the equation  $4x^2 - 24x + p = 0$  has no real roots. [1]

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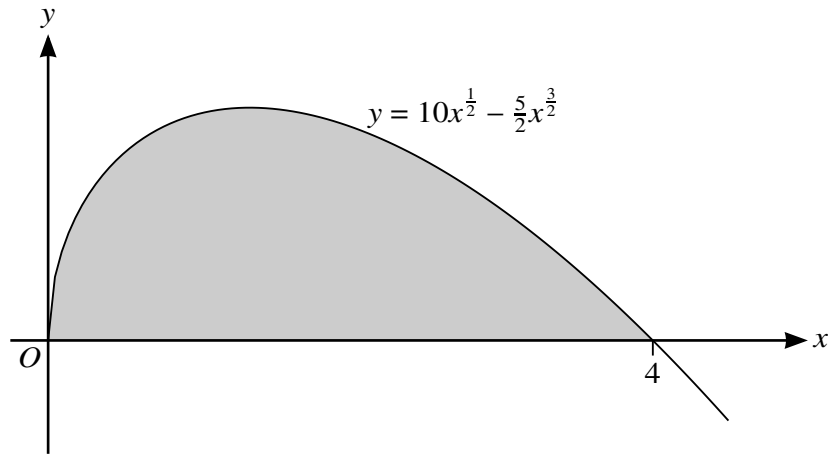
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The diagram shows the curve with equation  $y = 10x^{1/2} - \frac{5}{2}x^{3/2}$  for  $x > 0$ . The curve meets the  $x$ -axis at the points  $(0, 0)$  and  $(4, 0)$ .

Find the area of the shaded region.

[4]

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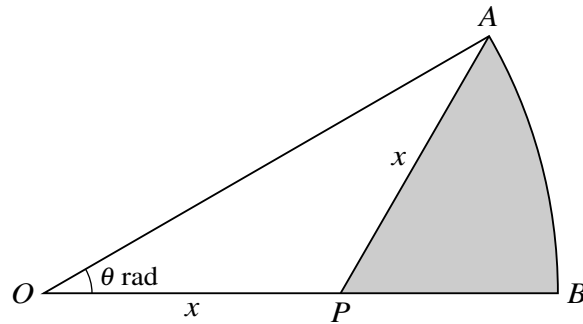
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The diagram shows a sector  $OAB$  of a circle with centre  $O$ . Angle  $AOB = \theta$  radians and  $OP = AP = x$ .

- (a) Show that the arc length  $AB$  is  $2x\theta \cos \theta$ . [2]

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- (b) Find the area of the shaded region  $APB$  in terms of  $x$  and  $\theta$ . [4]

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7 (a) (i) By first expanding  $(\cos \theta + \sin \theta)^2$ , find the three solutions of the equation

$$(\cos \theta + \sin \theta)^2 = 1$$

for  $0 \leq \theta \leq \pi$ .

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(ii) Hence verify that the only solutions of the equation  $\cos \theta + \sin \theta = 1$  for  $0 \leq \theta \leq \pi$  are 0 and  $\frac{1}{2}\pi$ . [2]

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- (b) Prove the identity  $\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} \equiv \frac{\cos \theta + \sin \theta - 1}{1 - 2 \sin^2 \theta}$ . [3]

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- (c) Using the results of (a)(ii) and (b), solve the equation

$$\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} = 2(\cos \theta + \sin \theta - 1)$$

for  $0 \leq \theta \leq \pi$ .

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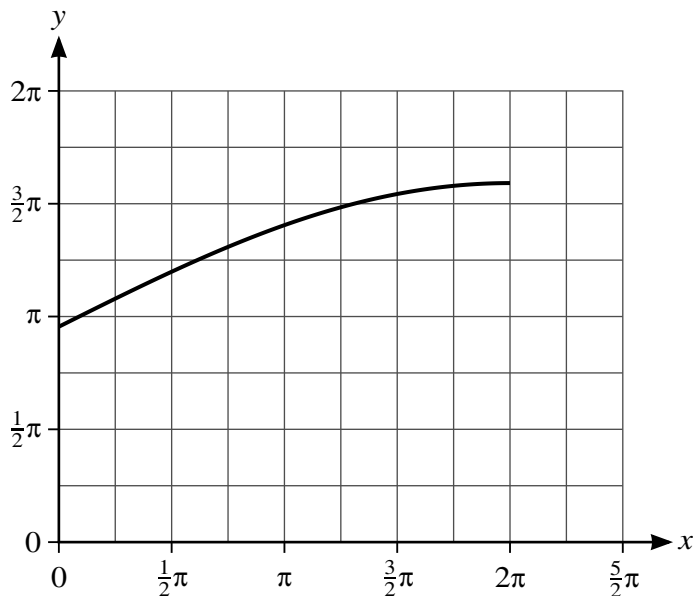
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The diagram shows the graph of  $y = f(x)$  where the function  $f$  is defined by

$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

(a) On the diagram above, sketch the graph of  $y = f^{-1}(x)$ . [2]

(b) Find an expression for  $f^{-1}(x)$ . [2]

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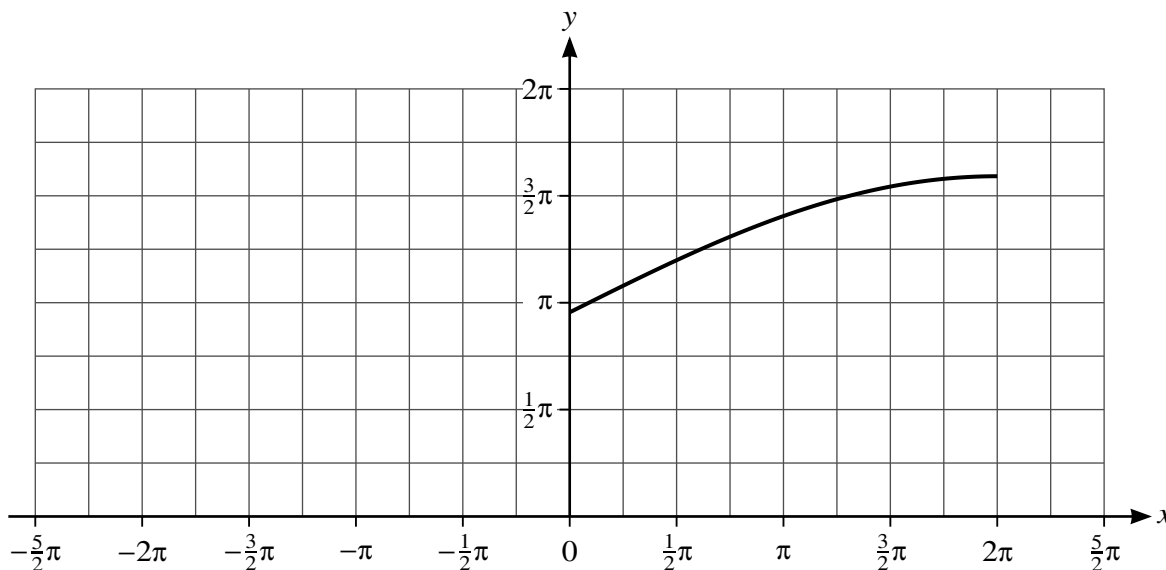
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(c)



The diagram above shows part of the graph of the function  $g(x) = 3 + 2 \sin \frac{1}{4}x$  for  $-2\pi \leq x \leq 2\pi$ .

Complete the sketch of the graph of  $g(x)$  on the diagram above and hence explain whether the function  $g$  has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of  $y = \sin x$  for  $0 \leq x \leq \frac{1}{2}\pi$  to the graph of  $y = f(x)$ , making clear the order in which the transformations are applied. [6]

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**9** The second term of a geometric progression is 16 and the sum to infinity is 100.

**(a)** Find the two possible values of the first term.

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- (b) Show that the  $n$ th term of one of the two possible geometric progressions is equal to  $4^{n-2}$  multiplied by the  $n$ th term of the other geometric progression. [4]

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**10** The equation of a circle is  $(x - a)^2 + (y - 3)^2 = 20$ . The line  $y = \frac{1}{2}x + 6$  is a tangent to the circle at the point  $P$ .

**(a)** Show that one possible value of  $a$  is 4 and find the other possible value. [5]

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- (b) For  $a = 4$ , find the equation of the normal to the circle at  $P$ . [4]

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- (c) For  $a = 4$ , find the equations of the two tangents to the circle which are parallel to the normal found in (b). [4]

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11 The equation of a curve is

$$y = k\sqrt{4x + 1} - x + 5,$$

where  $k$  is a positive constant.

(a) Find  $\frac{dy}{dx}$ . [2]

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(b) Find the  $x$ -coordinate of the stationary point in terms of  $k$ . [2]

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**Additional Page**

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